

MATH 110

(1) $a < f(x) < b$ (where $f(x)$ is a polynomial of degree one) :
add, subtract, multiply $\Rightarrow k_1 < x < k_2$ (see exp(2))

(2) $a < f(x) < b$ (i)

(where $f(x)$ is a polynomial of degree greater than one)

solve $\begin{cases} a < f(x) \Rightarrow x \in A \\ f(x) < b \Rightarrow x \in B \end{cases}$, the solution of (i) is $x \in A \cap B$.

(3) $g(x) < f(x) < h(x)$ (ii)

{where at least one of the functions $g(x), h(x)$ is not constant} :-

solve $\begin{cases} g(x) < f(x) \Rightarrow x \in A \\ f(x) < h(x) \Rightarrow x \in B \end{cases}$, the solution of (ii) is $x \in A \cap B$. (exc. 23)

(4) $a < |f(x)| < b$ (iii) **solve** $\begin{cases} a < |f(x)| \Rightarrow x \in A \\ |f(x)| < b \Rightarrow x \in B \end{cases}$,

the solution of (iii) is $x \in A \cap B$. (exc. 55)

(5) $f(x) < b$, where $f(x)$ is rational function $\Rightarrow f(x) - b < 0$, for exp

$$\frac{2x}{x-1} \leq 1 \Rightarrow \frac{2x}{x-1} - 1 \leq 0 \Rightarrow \frac{x+1}{x-1} \leq 0 \Rightarrow x+1=0, x-1=0 \Rightarrow x=-1, x=1$$

interval	$x+1$	$x-1$	$(x+1)(x-1)$
$x < -1$	-	-	+
$-1 < x < 1$	+	-	-
$x > 1$	+	+	+

$$\Rightarrow x \in [-1, 1)$$