Workshop Solutions to Sections 2.1 and 2.2

1) Find the domain of the function $f(x) = 9 - x^2$.

Solution:

Since $f(x)$ is a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

Note: The domain of any polynomial is $\mathbb{R}$.

2) Find the range of the function $f(x) = 9 - x^2$.

Solution:

$$R_f = (-\infty, 9]$$

3) Find the domain of the function $f(x) = 6 - 2x$.

Solution:

Since $f(x)$ is a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

4) Find the range of the function $f(x) = 6 - 2x$.

Solution:

Since $f(x)$ is a polynomial of degree one ($i.e.$ is of an odd degree), then

$$R_f = \mathbb{R} = (-\infty, \infty)$$

5) Find the domain of the function $f(x) = x^2 - 2x - 3$.

Solution:

Since $f(x)$ is a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

6) Find the domain of the function $f(x) = 1 + 2x^3 - x^5$.

Solution:

Since $f(x)$ is a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

7) Find the domain of the function $f(x) = 5$.

Solution:

Since $f(x)$ is a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

8) Find the range of the function $f(x) = 5$.

Solution:

$$R_f = \{5\}$$

9) Find the domain of the function $f(x) = |x - 1|$.

Solution:

Since $f(x)$ is an absolute value of a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

Note: The domain of an absolute value of any polynomial is $\mathbb{R}$.

10) Find the domain of the function $f(x) = |x + 5|$.

Solution:

Since $f(x)$ is an absolute value of a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

11) Find the domain of the function $f(x) = |x|$.

Solution:

Since $f(x)$ is an absolute value of a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

12) Find the range of the function $f(x) = |x|$.

Solution:

$$R_f = [0, \infty)$$

Note: The range of an absolute value of any polynomial is always $[0, \infty)$.

13) Find the domain of the function $f(x) = |3x - 6|$.

Solution:

Since $f(x)$ is an absolute value of a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

14) Find the domain of the function $f(x) = |9 - 3x|$.

Solution:

Since $f(x)$ is an absolute value of a polynomial, then

$$D_f = \mathbb{R} = (-\infty, \infty)$$

15) Find the domain of the function

$$f(x) = \frac{x + 2}{x - 3}$$

Solution:

$f(x)$ is defined when $x - 3 \neq 0 \implies x \neq 3$. So,

$$D_f = \mathbb{R} \setminus \{3\} = (-\infty, 3) \cup (3, \infty)$$

16) Find the domain of the function

$$f(x) = \frac{x - 2}{x + 3}$$

Solution:

$f(x)$ is defined when $x + 3 \neq 0 \implies x \neq -3$. So,

$$D_f = \mathbb{R} \setminus \{-3\} = (-\infty, -3) \cup (-3, \infty)$$

17) Find the domain of the function

$$f(x) = \frac{x+2}{x^2-9}$$

Solution:

$f(x)$ is defined when $x^2 - 9 \neq 0 \Longrightarrow x^2 \neq 9 \Longrightarrow x \neq \pm 3$. So,

$$D_f = \mathbb{R} \setminus \{-3,3\} = (-\infty,-3) \cup (-3,3) \cup (3,\infty)$$

18) Find the domain of the function

$$f(x) = \frac{x+2}{x^2-5x+6}$$

Solution:

$f(x)$ is defined when $x^2 - 5x + 6 \neq 0$

$\Longrightarrow (x-2)(x-3) \neq 0 \Longrightarrow x \neq 2$ or $x \neq 3$. So,

$$D_f = \mathbb{R} \setminus \{2,3\} = (-\infty,2) \cup (2,3) \cup (3,\infty)$$

19) Find the domain of the function

$$f(x) = \frac{x+2}{x^2-x-6}$$

Solution:

$f(x)$ is defined when $x^2 - x - 6 \neq 0$

$\Longrightarrow (x+2)(x-3) \neq 0 \Longrightarrow x \neq -2$ or $x \neq 3$. So,

$$D_f = \mathbb{R} \setminus \{-2,3\} = (-\infty,-2) \cup (-2,3) \cup (3,\infty)$$

20) Find the domain of the function

$$f(x) = \frac{x+2}{x^2+9}$$

Solution:

$f(x)$ is defined when $x^2 + 9 \neq 0$ but for any value x the denominator $x^2 + 9$ cannot be 0. So,

$$D_f = \mathbb{R} = (-\infty,\infty)$$

21) Find the domain of the function

$$f(x) = \sqrt[3]{x-3}$$

Solution:

$$D_f = \mathbb{R} = (-\infty,\infty)$$

Note: The domain of an odd root of any polynomial is $\mathbb{R}$.

22) Find the domain of the function

$$f(x) = \sqrt{x-3}$$

Solution:

$f(x)$ is defined when $x - 3 \geq 0 \Longrightarrow x \geq 3$ because $f(x)$ is an even root. So,

$$D_f = [3,\infty)$$

23) Find the domain of the function

$$f(x) = \sqrt{3-x}$$

Solution:

$f(x)$ is defined when $3 - x \geq 0 \Longrightarrow -x \geq -3 \Longrightarrow x \leq 3$ because $f(x)$ is an even root. So,

$$D_f = (-\infty,3]$$

24) Find the domain of the function

$$f(x) = \sqrt{x+3}$$

Solution:

$f(x)$ is defined when $x + 3 \geq 0 \Longrightarrow x \geq -3$ because $f(x)$ is an even root. So,

$$D_f = [-3,\infty)$$

25) Find the domain of the function

$$f(x) = \sqrt{-x}$$

Solution:

$f(x)$ is defined when $-x \geq 0 \Longrightarrow x \leq 0$ because $f(x)$ is an even root. So,

$$D_f = (-\infty,0]$$

26) Find the range of the function

$$f(x) = \sqrt{-x}$$

Solution:

$$R_f = [0,\infty)$$

Note: The range of an even root is always ≥ 0 .

27) Find the domain of the function

$$f(x) = \sqrt{9-x^2}$$

Solution:

$f(x)$ is defined when $9 - x^2 \geq 0 \Longrightarrow -x^2 \geq -9 \Longrightarrow$

$x^2 \leq 9 \Longrightarrow \sqrt{x^2} \leq \sqrt{9} \Longrightarrow |x| \leq 3 \Longrightarrow -3 \leq x \leq 3$. So,

$$D_f = [-3,3]$$

28) Find the domain of the function

$$f(x) = \frac{x+2}{\sqrt{x-3}}$$

Solution:

$f(x)$ is defined when $x - 3 > 0 \Longrightarrow x > 3$. So,

$$D_f = (3,\infty)$$

29) Find the domain of the function

$$f(x) = \frac{x+2}{\sqrt{9-x^2}}$$

Solution:

$f(x)$ is defined when $9 - x^2 > 0 \Longrightarrow -x^2 > -9$

$\Longrightarrow x^2 < 9 \Longrightarrow \sqrt{x^2} < \sqrt{9} \Longrightarrow |x| < 3 \Longrightarrow -3 < x < 3$. So,

$$D_f = (-3,3)$$

30) Find the domain of the function

$$f(x) = \sqrt{x^2-9}$$

Solution:

$f(x)$ is defined when $x^2 - 9 \geq 0 \Longrightarrow x^2 \geq 9$

$\Longrightarrow \sqrt{x^2} \geq \sqrt{9} \Longrightarrow |x| \geq 3 \Longrightarrow x \geq 3$ or $x \leq -3$. So,

$$D_f = (-\infty,-3] \cup [3,\infty)$$

31) Find the range of the function

$$f(x) = \sqrt{x^2 - 9}$$

<u>Solution:</u>

$$R_f = [0, \infty)$$

32) Find the domain of the function

$$f(x) = \frac{x+2}{\sqrt{x^2 - 9}}$$

<u>Solution:</u>

$f(x)$ is defined when $x^2 - 9 > 0 \implies x^2 > 9$

$\implies \sqrt{x^2} > \sqrt{9} \implies |x| > 3 \implies x > 3$ or $x < -3$.

So,

$$D_f = (-\infty, -3) \cup (3, \infty)$$

33) Find the domain of the function

$$f(x) = \sqrt{9 + x^2}$$

<u>Solution:</u>

$f(x)$ is defined when $9 + x^2 \geq 0$ but it is always true for any value x . So,

$$D_f = \mathbb{R}$$

34) Find the domain of the function

$$f(x) = \sqrt[4]{x^2 - 25}$$

<u>Solution:</u>

$f(x)$ is defined when $x^2 - 25 \geq 0 \implies x^2 \geq 25$

$\implies \sqrt{x^2} \geq \sqrt{25} \implies |x| \geq 5 \implies x \geq 5$ or $x \leq -5$.

So,

$$D_f = (-\infty, -5] \cup [5, \infty)$$

35) Find the domain of the function

$$f(x) = \sqrt[6]{16 - x^2}$$

<u>Solution:</u>

$f(x)$ is defined when $16 - x^2 \geq 0 \implies -x^2 \geq -16 \implies$

$x^2 \leq 16 \implies \sqrt{x^2} \leq \sqrt{16} \implies |x| \leq 4 \implies -4 \leq x \leq 4$.

So,

$$D_f = [-4,4]$$

36) Find the range of the function

$$f(x) = \sqrt{16 - x^2}$$

<u>Solution:</u>

We know that $f(x)$ is defined when $16 - x^2 \geq 0$

$\implies -x^2 \geq -16 \implies x^2 \leq 16 \implies \sqrt{x^2} \leq \sqrt{16}$

$\implies |x| \leq 4 \implies -4 \leq x \leq 4$. So,

$$D_f = [-4,4]$$

Using D_f we find the outputs vary from 0 to 4 . Hence,

$$R_f = [0,4]$$

37) Find the domain of the function

$$f(x) = \frac{x + |x|}{x}$$

<u>Solution:</u>

$f(x)$ is defined when $x \neq 0$. So,

$$D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$$

38) Find the domain of the function

$$f(x) = \begin{cases} -\frac{1}{x}, & x < 0 \\ x, & x \geq 0 \end{cases}$$

<u>Solution:</u>

It is clear from the definition of the function $f(x)$ that

$$D_f = \mathbb{R} = (-\infty, \infty)$$

39) Find the domain of the function

$$f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$$

<u>Solution:</u>

$f(x)$ is defined when

1- $x \geq 0 \quad \implies \quad D_{\sqrt{x}} = [0, \infty)$

2- $x^2 + 1 > 0$ but this is always true for all x

$\implies D_{\sqrt{x^2+1}} = \mathbb{R}$.

Hence,

$$D_f = D_{\sqrt{x}} \cap D_{\sqrt{x^2+1}} = [0, \infty) \cap \mathbb{R} = [0, \infty)$$

40) Find the domain of the function

$$f(x) = \sqrt{x - 1} + \sqrt{x + 3}$$

<u>Solution:</u>

$f(x)$ is defined when

1- $x - 1 \geq 0 \implies x \geq 1 \implies D_{\sqrt{x-1}} = [1, \infty)$

2- $x + 3 \geq 0 \implies x \geq -3 \implies D_{\sqrt{x+3}} = [-3, \infty)$

Hence,

$$D_f = D_{\sqrt{x-1}} \cap D_{\sqrt{x+3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$$

41) The function $f(x) = 3x^4 + x^2 + 1$ is a polynomial function.

42) The function $f(x) = 5x^3 + x^2 + 7$ is a cubic function.

43) The function $f(x) = -3x^2 + 7$ is a quadratic function.

44) The function $f(x) = 2x + 3$ is a linear function.

45) The function $f(x) = x^7$ is a power function.

46) The function $f(x) = \frac{2x+3}{x^2-1}$ is a rational function.

47) The function $f(x) = \frac{x-3}{x+2}$ is a rational function and we can say it is an algebraic function as well.

48) The function $f(x) = \sin x$ is a trigonometric function.

49) The function $f(x) = e^x$ is a natural exponential function.

50) The function $f(x) = 3^x$ is a general exponential function.

51) The function $f(x) = x^2 + \sqrt{x-2}$ is an algebraic function.

52) The function $f(x) = -3$ is a constant function.

53) The function $f(x) = \log_3 x$ is a general logarithmic function.

54) The function $f(x) = \ln x$ is a natural logarithmic function.

55) The function $f(x) = 3x^4 + x^2 + 1$ is

Solution:

$$f(-x) = 3(-x)^4 + (-x)^2 + 1 = 3x^4 + x^2 + 1 = f(x)$$

Hence,
$f(x)$ is an even function.

56) The function $f(x) = 9 - x^2$ is

Solution:

$$f(-x) = 9 - (-x)^2 = 9 - x^2 = f(x)$$

Hence,
$f(x)$ is an even function.

57) The function $f(x) = x^5 - x$ is

Solution:

$$f(-x) = (-x)^5 - (-x) = -x^5 + x$$
$$= -(x^5 - x) = -f(x)$$

Hence,
$f(x)$ is an odd function.

58) The function $f(x) = 2 - \sqrt[5]{x}$ is

Solution:

$$f(-x) = 2 - \sqrt[5]{(-x)} = 2 - \sqrt[5]{-x} = 2 + \sqrt[5]{x}$$
$$= -\left(-2 - \sqrt[5]{x}\right)$$

Hence,
$f(x)$ is neither even nor odd.

59) The function $f(x) = 3x + \frac{2}{\sqrt{x^2+9}}$ is

Solution:

$$f(-x) = 3(-x) + \frac{2}{\sqrt{(-x)^2 + 9}} = -3x + \frac{2}{\sqrt{x^2+9}}$$
$$= -\left(3x - \frac{2}{\sqrt{x^2+9}}\right)$$

Hence,
$f(x)$ is neither even nor odd.

60) The function $f(x) = \frac{3}{\sqrt{x^2+9}}$ is

Solution:

$$f(-x) = \frac{3}{\sqrt{(-x)^2+9}} = \frac{3}{\sqrt{x^2+9}} = f(x)$$

Hence,
$f(x)$ is an even function.

61) The function $f(x) = \sqrt{4 + x^2}$ is

Solution:

$$f(-x) = \sqrt{4 + (-x)^2} = \sqrt{4 + x^2} = f(x)$$

Hence,
$f(x)$ is an even function.

62) The function $f(x) = 3$ is

Solution:

Since the graph of the constant function 3 is symmetric about the $y - axis$, then
$f(x)$ is an even function.

63) The function $f(x) = \frac{9-x^2}{x-2}$ is

Solution:

$$f(-x) = \frac{9 - (-x)^2}{(-x) - 2} = \frac{9 - x^2}{-x - 2}$$
$$= -\left(\frac{9 - x^2}{x + 2}\right)$$

Hence,
$f(x)$ is neither even nor odd.

64) The function $f(x) = \frac{x^2-4}{x^2+1}$ is

Solution:

$$f(-x) = \frac{(-x)^2 - 4}{(-x)^2 + 1} = \frac{x^2 - 4}{x^2 + 1} = f(x)$$

Hence,
$f(x)$ is an even function.

65) The function $f(x) = 3|x|$ is

Solution:

$$f(-x) = 3|(-x)| = 3|x| = f(x)$$

Hence,
$f(x)$ is an even function.

66) The function $f(x) = x^{-2}$ is

Solution:

$$f(x) = x^{-2} = \frac{1}{x^2}$$
$$f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = f(x)$$

Hence, $f(x)$ is an even function.

67) The function $f(x) = x^3 - 2x + 5$ is <u>Solution:</u> $f(-x) = (-x)^3 - 2(-x) + 5 \quad = -x^3 + 2x + 5$ $= -(x^3 - 2x - 5)$ Hence, $f(x)$ is neither even nor odd.	68) The function $f(x) = \sqrt[3]{x^5} - x^3 + x$ is <u>Solution:</u> $f(-x) = \sqrt[3]{(-x)^5} - (-x)^3 + (-x) \; = -\sqrt[3]{x^5} + x^3 - x$ $= -\left(\sqrt[3]{x^5} - x^3 + x\right) = -f(x)$ Hence, $f(x)$ is an odd function.
69) The function $f(x) = 7$ is <u>Solution:</u> Since the graph of the constant function 7 is symmetric about the $y - axis$, then $f(x)$ is an even function.	70) The function $f(x) = \frac{x^3-4}{x^3+1}$ is <u>Solution:</u> $f(-x) = \dfrac{(-x)^3 - 4}{(-x)^3 + 1} = \dfrac{-x^3 - 4}{-x^3 + 1} = -\dfrac{x^3 + 4}{-x^3 + 1}$ Hence, $f(x)$ is neither even nor odd.
71) The function $f(x) = \frac{x^2-1}{x^3+3}$ is <u>Solution:</u> $f(-x) = \dfrac{(-x)^2 - 1}{(-x)^3 + 3} = \dfrac{x^2 - 1}{-x^3 + 3} = -\dfrac{x^2 - 1}{x^3 - 3}$ Hence, $f(x)$ is neither even nor odd.	72) The function $f(x) = x^6 - 4x^2 + 1$ is <u>Solution:</u> $f(-x) = (-x)^6 - 4(-x)^2 + 1 = x^6 - 4x^2 + 1 = f(x)$ Hence, $f(x)$ is an even function.
73) The function $f(x) = x^2$ is increasing on $(0, \infty)$.	74) The function $f(x) = x^2$ is decreasing on $(-\infty, 0)$.
75) The function $f(x) = x^3$ is increasing on $(-\infty, \infty)$.	76) The function $f(x) = x^3$ is not decreasing at all.
77) The function $f(x) = \sqrt{x}$ is increasing on $(0, \infty)$.	78) The function $f(x) = \sqrt{x}$ is not decreasing at all.
79) The function $f(x) = \frac{1}{x}$ is not increasing at all.	80) The function $f(x) = \frac{1}{x}$ is decreasing on $(-\infty, \infty)$.