math 464

Second Homework

Due Date Sunday 18/ 6 / 1437, at 11:55 Pm.

Name: **Number:**

Always try to justify your answer (SHORT PROOF).

Q1: Prove or disprove:

(a) Every topology has a Subbase.

(b) If $\beta^{/}$ is a base for the topological space (X, τ) and $\beta^{/} \subset \beta$. Then β is a base for τ.

Q2: Let X be any set which has more than one element. Fix an element $p \in X$. Define $\mathcal{T}_p \subset \mathcal{P}(X)$ as follows:

$$\mathcal{T}_p = \{\emptyset\} \cup \{W \subseteq X \,:\, p \in W\}.$$

Check that $\mathcal{T}_p$ is a topology on X. $\mathcal{T}_p$ is called *the particular point* topology on X.

Q3: Consider the lower limit topology τ on $\mathbb{R}$ which has

$\beta = \{[a, b) : a, b \in \mathbb{R}; a < b\}$ as its base. Show that $[1, 7)$ is τ-clopen set ?

Q4: Let $X = \{a, b, c, d, e, f\}$, and $S = \{\{a\}, \{a, b\}, \{b, c\}, \{c, d\}, \{d, e\}, \{e, f\}, \{f\}\}$ is a subbasis for the topology τ on X. What is τ?

[Classify τ(kind and members)]

Good Luck :)